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ABSTRACT

We develop a model of statistical discrimination in occupational licensing. In the model, there is endogenous occupation selection and wage determination that depends on how costly it is to obtain the license and the productivity of the human capital that is bundled with the license. Under these assumptions, we find a unique equilibrium with sharp comparative statics for the licensing premiums. The key theoretical result in this paper is that the licensing premium is higher for workers who are members of demographic groups that face a higher cost of licensing. The intuition for this result is that the higher cost of licensing makes the license a more informative labor market signal. (This is a similar insight to Spence 1973). The predictions of the model can explain, for example, the empirical finding in the literature that occupational licenses that preclude felons close the racial wage gap among men by conferring a higher premium to black men than to white men (Blair and Chung 2018). Moreover, we show that in general the optimal cost of licensing is nonzero: an infinitely costly licenses screens out all workers, while a costless license is no screen at all.

JEL Classification Codes: D45, D82, D83, J44

Key Words: statistical discrimination, occupational licensing, labor market signal, equilibrium

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1 Discrimination & Licensing

Statistical discrimination against female and minority workers occurs when employers believe that workers of these types are less qualified, on average, or have a productivity that is drawn from a noisier distribution than their male or white peers, respectively (Arrow, 1973). For example, if employers believe that women have lower (higher) expected productivity than that of white men, in equilibrium, women will receive lower (higher) wages than white men if individual productivity is not directly observable.[2] Likewise, holding expected productivity constant across groups, if employers believe that the productivity distribution of women is noisier (sharper) than that of white men, in equilibrium, women will receive lower (higher) wages than white men. An important theoretical finding in the literature is that employer priors about worker productivity by worker type can themselves endogenously generate wage gaps through a self-fulfilling prophecy mechanism, even if the priors are incorrect at the outset (Coate and Loury, 1993).

In the spirit of Coate and Loury (1993) and Moro and Norman (2004), we develop a model of occupational licensing in which there is endogenous occupation selection and endogenous wage determination. Our model differs from these two self-fulfilling prophecy models in that we assume that the distribution of worker ability is heterogeneous by worker type, fully known to employers, and correctly perceived by employers *ex ante*. The assumptions that we make allow for a unique equilibrium wage for licensed and unlicensed workers for each race and gender group.

Our model admits a unique equilibrium, in contrast to discrimination models with a self-fulfilling prophecy feature, which admit multiple equilibria.[3] We are expressly interested in the comparative statics of the model that reflect whether occupational licensing

[2] The initial wages will be lower, but over time, as the firm learns about worker productivity from on-the-job performance, these wage gaps should diminish, as in Altonji and Pierret (2001).

[3] The existence of multiple equilibria is a feature of self-fulfilling prophecy models. This demonstrates how inequality in labor market outcomes can arise between two *ex ante* identical groups of workers. By comparison, our goal in this paper is to take firm beliefs about workers' abilities as given and determine the extent to which workers sort into licensed occupations in order to signal their type.

results in heterogeneous wage premium for licensed workers by race or gender. It is either because firms have different priors over the underlying distributions of ability, or because workers face different average costs of investing in the licensing signal.

Our work also contributes to the theoretical literature on occupational licensing by providing an analytically tractable model of licensing as a job market signal, in the spirit of Spence (1973). The standard model of occupational licensing is Leland (1979), which studied licensing from an optimal legislation vantage point. Whereas Leland (1979) focused on whether it is socially optimal to have quality standards, Persico (2015) studied the incentives of incumbent workers to impose occupational requirements on new entrants. We build a micro-founded model in which the licensing decisions of workers and the wages offered by firms are endogenous outcomes of a two-period sequential screening game played by firms and workers.

2 Model

Our model is a two-sector, two-period model of firms and workers, consisting of a unit measure of risk-neutral workers and an occupational licensing requirement for workers in sector 1 but not sector 2. In each sector, there is a single representative firm. Firms *do not* observe a worker's ability, but firms do observe whether or not a worker has a license. Because licensing is costly and more easily accessible for workers of higher ability, an occupational license acts as a signal of ability in an analogous way to education in Spence (1973).

In period 1, firms set wages to maximize profits, namely ω_L for the licensed sector and ω_U for the unlicensed sector. In period 2, workers choose the sector that delivers the highest utility, given the wages offered by firms and given the relative preferences of workers over employment in the two sectors. The equilibrium of the model is a vector of wages (ω_L^*, ω_U^*) and a fraction of licensed workers f^* that satisfies the utility maximization motive of workers and the profit-maximizing motive of firms. Because firms, which are the uninformed party in our model, move first, our model falls under the technical definition of a screening model (Stiglitz and Weiss, 1990).

2.1 Description of Workers' Tastes and Abilities

Each worker, indexed by the subscript i, is endowed with an ability a_i and a relative taste for the unlicensed sector ϵ_i. For licenses that preclude ex-offenders, ability maps to criminal history in the model. For licenses that require passing a test, ability maps to cognitive ability. In cases where the license has no test or criminal requirement, ability measures the ability/willingness to pay the hassle cost of completing the licensing paperwork.

The ability type and the relative sector preference are independently and identically distributed across workers and drawn from the following two uniform distributions: $a_i \sim U[\mu_a - \sigma_a, \mu_a + \sigma_a]$ and $\epsilon_i \sim U[\mu_\epsilon - \sigma_\epsilon, \mu_\epsilon + \sigma_\epsilon]$. We assume uniform distributions for the sake of analytical tractability. The sector taste parameters, μ_ϵ and σ_ϵ, are measured in units of dollars so that they enter the worker's utility function on the same footing as wages. The ability and preference distribution are allowed to be different for workers of different racial and gender groups. For notational simplicity, however, we suppress the group index and solve the model separately for each group. The sign of the comparative statics will describe how differences in firms' priors between women and men, minorities and nonminorities, map onto differences in the licensing wage premium across demographic groups.

Obtaining an occupational license is costly for workers of all abilities. To obtain an occupational license, a worker of ability a_i incurs a cost:

$$c(a_i) = c_0 - \theta(a_i - \mu_a). \quad (1)$$

The parameter $c_0 > 0$ is the unconditional average cost of obtaining an occupational license for workers of a given group.[4] For example, the average cost of obtaining a license in an occupation with a felony restriction will be higher on average for workers from groups that face higher incarceration rates. The parameter θ is the marginal benefit of ability. Each unit increase in ability lowers the cost of licensing by an amount θ. For ability measures that make it easier for a worker to obtain an occupational license (*e.g.*, IQ), we will assume a positive marginal benefit of ability (*i.e.*, $\theta > 0$). For ability measures such as a worker's level of criminality or criminal history, which, by law, make obtaining an occupational license more difficult, we assume a negative marginal benefit of ability (*i.e.*, $\theta < 0$).

In the unlicensed sector, a worker i receives utility $V_{U,i}$, which is the sum of the wages earned in the unlicensed sector, ω_U, and the relative taste that she has for the unlicensed sector, ϵ_i:

$$V_{U,i} = \omega_U + \epsilon_i. \tag{2}$$

In the licensed sector, a worker i receives utility $V_{L,i}$, which is the difference between the wages earned in the licensed sector, ω_L, and the cost, $c(a_i)$, that she incurred in order to obtain the license:

$$V_{L,i} = \omega_L - [c_0 - \theta(a_i - \mu_a)]. \tag{3}$$

2.2 Firms

Each firm, j, possesses a technology that converts one unit of worker ability into $\bar{\omega}$ dollars' worth of goods. In the licensed sector, $j = 1$, the occupational license is also bundled with an exogenous level of useful human capital (training) $0 \leq h \leq 1$, which augments the worker's ability to utilize the technology by a factor of $(1 + h)$.[5] The expected profit for

[4] It is also the cost of licensing for the worker of average ability, $a_i = \mu_a$.

[5] The cost of acquiring this human capital is borne by the workers, as in Equation (1).

the representative firm in the *licensed* occupation is given by

$$E[\pi_1] = \underbrace{\overbrace{\bar{\omega}(1+h)E[a_i|L_i=1]}^{\text{Avg. Output per Worker}}\overbrace{E[P(L_i=1|a_i)]}^{\text{Measure of Workers}}}_{\text{Expected Revenue}} - \underbrace{\omega_L E[P(L_i=1|a_i)]}_{\text{Expected Labor Cost}}, \tag{4}$$

where $E[a_i|L_i = 1]$ is the expected ability of a worker conditional on employment in the licensed sector and $E[P(L_i = 1|a_i)]$ is the fraction of workers in the licensed sector. The expected profit for the representative firm in the *unlicensed* occupation is given by

$$E[\pi_2] = \underbrace{\bar{\omega}E[a_i|L_i=0]E[P(L_i=0|a_i)]}_{\text{Expected Revenue}} - \underbrace{\omega_U E[P(L_i=0|a_i)]}_{\text{Expected Labor Cost}}, \tag{5}$$

where $E[a_i|L_i = 0]$ is the expected ability of a worker conditional on employment in the unlicensed sector and $E[P(L_i = 0|a_i)]$ is the fraction of workers employed in the unlicensed sector.

Proposition 1. *If the average cost of licensing $c_0 \in (\underline{c}, \bar{c})$, where $\underline{c} \equiv h\bar{\omega}\mu_a - \mu_\epsilon - 3\sigma_\epsilon$ and $\bar{c} \equiv h\bar{\omega}\mu_a - \mu_\epsilon + 3\sigma_\epsilon$, a unique subgame perfect Nash equilibrium exists with a wage for unlicensed workers:*

$$\omega_U^* = \bar{\omega}\mu_a - \frac{1}{3}(c_0 - \underline{c}), \tag{6}$$

a wage for licensed workers:

$$\omega_L^* = \underbrace{\bar{\omega}\mu_a - \frac{1}{3}(c_0 - \underline{c})}_{\omega_U^*} + \underbrace{\frac{1}{3}h\bar{\omega}\mu_a + \frac{2}{3}(c_0 + \mu_\epsilon)}_{\textit{Wage Benefit of Licensing}}, \tag{7}$$

and the fraction of workers with an occupational license is an interior point given by

$$f^* \equiv E[P(L_i = 1|a_i)] = \left(\frac{\bar{c} - c_0}{6\sigma_\epsilon}\right). \tag{8}$$

Proof. See Appendix. □

If $c_0 \geq \bar{c}$, it is not worthwhile to have a license even for the highest-ability workers. Hence, all workers pool on not having a license, *i.e.*, $f^* = 0$. If the cost of licensing is sufficiently low, *i.e.*, $c_0 \leq \underline{c}$, licensing is cost-effective even for the lowest ability type, and all workers pool on having a license, *i.e.*, $f^* = 1$. In between these two extremes, we have an interior solution in which a fraction, $0 < f^* < 1$, of the workers select into the licensed sector.

Proposition 2. *The licensing premium, α, is unambiguously increasing in the average cost of the license, i.e.* $\frac{d\alpha}{dc_0} > 0$.

Proof. The licensing premium α is defined as

$$\alpha \equiv \frac{\omega_L^* - \omega_U^*}{\omega_U^*} \tag{9}$$

$$\alpha = \frac{\frac{1}{3}\bar{\omega}\mu_a h + \frac{2}{3}(c_0 + \mu_\epsilon)}{\left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}(c_0 + \mu_\epsilon) - \sigma_\epsilon}. \tag{10}$$

$$\frac{d\alpha}{dc_0} = \frac{1}{3}\left(\frac{\omega_L^* - \omega_U^*}{{\omega_U^*}^2}\right) > 0. \tag{11}$$

□

The higher the cost of licensing, the costlier it is for lower-ability workers to obtain the license, and hence the stronger the signaling value of the occupational license. In an earlier paper of ours, we regard felony restrictions on a license as imposing a cost that differentially affects black men—the group facing a higher incarceration rate (Blair and Chung, 2018). Our current model provides a theoretical basis for the main finding in that paper: that a higher licensing premium for black men (relative to white men) only exists in the licensed occupations with a permanent felony restriction. This model also builds on the empirical findings of Law and Marks (2009), which demonstrated that licensing laws during the Progressive Era increased racial diversity in the teaching and practical nursing professions and increased gender diversity in the fields of engineering and pharmacy.

Proposition 3. *The licensing premium increases the level of human capital bundled with the license (h), if the licensing premium is less than 100%. The licensing premium unambiguously* decreases *the average ability of workers (μ_a).*

Proof. See Appendix. □

Intuitively, the more human capital that is bundled with the license, the higher the marginal product of labor, and hence the higher the equilibrium wage. Moreover, the license is more informative when the expected ability of the worker, given other observables—e.g., race and gender—is lower. Hence, the higher licensing premium.

In Blair and Chung (2018), we also show that the licensing premium for women (both black and white) is greater than the licensing premium for white men. This result is consistent with groups with lower perceived ability earning higher licensing premiums.

Proposition 4. *Define the* ***industry surplus*** *as the sum of firm profits and worker wages net of the licensing cost. The industry surplus is maximized by a nonnegative average cost of licensing:*

$$c_0^* = \frac{1}{2}\left(\bar{c} + h\bar{\omega}\mu_a\right). \tag{12}$$

Proof. See Appendix. □

The intuition for this result is similar to that in Spence (1973)—a license is informative because it is costly. In a market with workers of heterogeneous abilities, the optimal cost-of-licensing burden is neither zero nor infinity.

One important caveat here is that the industry surplus differs from the typical social surplus in that it abstracts from the welfare loss experienced by customers from higher prices, as in Kleiner and Soltas (2019). In this respect, this welfare calculation is closer in spirit to the producer surplus in Persico (2015), where the goal is to determine whether firms and incumbent workers, acting collusively, benefit from licensing, given that workers will endure the cost of licensing.

3 Conclusion

Economists traditionally viewed occupational licensing as a labor market friction (Friedman, 1962). The arguments in this paper suggest that licensing is also an informative labor market signal, because it is costly to obtain. Consistent with this model, in Blair and Chung (2018), we show that occupational licensing closes the racial wage gap between men when the license credibly signals a worker's criminal history. Given the intractable nature of the racial wage gap, as documented in Bayer and Charles (2018), it is remarkable that occupational licensing can close the racial wage gap. For a reform of occupational licensing to be successful, a key implication of our work is that policymakers need to acknowledge the signaling value of a license and its differentially positive impact on the wages of workers who would otherwise face statistical discrimination on the basis of race or gender.

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A Mathematical Appendix

A.1 Proof of Theorem 1

To solve this sequential game, we use the solution concept of subgame perfect equilibrium (SPE). In an SPE, we solve the model using backwards induction. First, workers in Period 2 sort into the sector that produces the highest net return, given wages and their preferences. Next, in Period 1, the representative firm in each sector chooses the corresponding wage to maximize firm profits, given the sorting of workers.

A.1.1 Period #2: Workers Choose Sector

Starting in Period 2, the probability that a worker of ability a_i sorts into the licensed sector, $P(L = 1|a_i)$, is given by the probability that the net benefit of working in the licensed sector is greater than the net benefit of working in the unlicensed sector:

$$\begin{aligned} P(L_i = 1|a_i) = \text{Prob}(V_{L,i} > V_{U,i}) &= \text{Prob}(\omega_L - c_0 - \omega_U + \theta(a_i - \mu_a) > \epsilon_i) \\ &= \frac{1}{2} + \frac{\Delta\omega + \theta(a_i - \mu_a)}{2\sigma_\epsilon}, \end{aligned} \tag{13}$$

where $\Delta\omega \equiv (\omega_L - c_0) - (\omega_U + \mu_\epsilon)$ is the expected net benefit of licensing across workers of all types. The conditional probability of licensing is increasing in the expected net benefit of licensing. It is also increasing in worker ability for cases where worker ability lowers the cost of licensing $\theta > 0$, but decreasing in worker ability in cases where worker ability increases the cost of licensing $\theta < 0$.

A.1.2 Period #1: Firms Choose Wages

Next, we must compute firm profits given the sorting decisions of workers. In order to compute profits for the representative firms in both the licensed and unlicensed sectors, we first compute the fraction of workers who sort into the licensed profession and the unlicensed profession, *i.e.*, $E[P(L_i = 1|a_i)]$ and $E[P(L_i = 0|a_i)]$, because these quantities enter into the expected labor cost of the firms:

$$E[P(L_i=1|a_i)] = \frac{1}{2\sigma_a}\int_{\mu_a-\sigma_a}^{\mu_a+\sigma_a} P(L_i=1|a_i)da_i = \frac{1}{2\sigma_a}\int_{\mu_a-\sigma_a}^{\mu_a+\sigma_a}\left[\frac{1}{2}+\frac{\Delta\omega+\theta(a_i-\mu_a)}{2\sigma_\epsilon}\right]da_i$$
$$= \frac{1}{2}+\frac{\Delta\omega}{2\sigma_\epsilon} \tag{14}$$

Given that we have a two-sector model, a worker is employed either in the licensed or in the unlicensed sector. Consequently,

$$\begin{aligned} E[P(L_i=0|a_i)] &= 1 - E[P(L_i=1|a_i)] \\ &= \frac{1}{2}-\frac{\Delta\omega}{2\sigma_\epsilon} \end{aligned} \tag{15}$$

To compute firm profits, we must also compute the expected ability level of a worker given that she has a license $E(a_i|L_i = 1)$ and given that she does not have a license $E(a_i|L_i = 0)$, both of which contribute to firm revenue:

$$\begin{aligned} E[a_i|L_i=1] = \int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i\frac{P(L_i=1|a_i)P(a_i)}{P(L_i=1)}da_i &= \frac{1}{2\sigma_a}\int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i\frac{\left[\frac{1}{2}+\frac{\Delta\omega+\theta(a_i-\mu_a)}{2\sigma_\epsilon}\right]}{\frac{1}{2}+\frac{\Delta\omega}{2\sigma_\epsilon}}da_i \\ &= \mu_a+\frac{\theta\sigma_a^2}{3(\sigma_\epsilon+\Delta\omega)} \end{aligned} \tag{16}$$

Similarly,

$$\begin{aligned} E[a_i|L_i=0] = \int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i\frac{P(L_i=0|a_i)P(a_i)}{P(L_i=0)}da_i &= \frac{1}{2\sigma_a}\int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i\frac{\left[\frac{1}{2}-\frac{\Delta\omega+\theta(a_i-\mu_a)}{2\sigma_\epsilon}\right]}{\frac{1}{2}-\frac{\Delta\omega}{2\sigma_\epsilon}}da_i \\ &= \mu_a-\frac{\theta\sigma_a^2}{3(\sigma_\epsilon-\Delta\omega)} \end{aligned} \tag{17}$$

Putting this all together, we get the results that profits in the licensed sector are given by

$$\pi_1 = \underbrace{\left((1+h)\bar{\omega}\left[\mu_a+\frac{\theta\sigma_a^2}{3(\sigma_\epsilon+\Delta\omega)}\right]-\omega_L\right)}_{\text{Expected Profit per. licensed worker}} \times \underbrace{\left[\frac{1}{2}+\frac{\Delta\omega}{2\sigma_\epsilon}\right]}_{\text{Frac. Licensed workers}}, \tag{18}$$

and firm profits in the unlicensed sector are given by

$$\pi_2 = \left(\bar{\omega}\left[\mu_a - \frac{\theta\sigma_a^2}{3(\sigma_\epsilon - \Delta\omega)}\right] - \omega_U\right)\left[\frac{1}{2} - \frac{\Delta\omega}{2\sigma_\epsilon}\right] \tag{19}$$

Firm 1 chooses ω_L to maximize its profits, π_1. This results in the following first-order condition, $\frac{\partial\pi_1}{\partial\omega_L} = 0$:

$$\underbrace{-\left(1 + \left[\frac{(1+h)\bar{\omega}\theta\sigma_a^2}{3(\sigma_\epsilon + \Delta\omega)^2}\right]\right)\left[\frac{1}{2} + \frac{\Delta\omega}{2\sigma_\epsilon}\right]}_{\text{Decrease in Unit Profit}} + \underbrace{\frac{1}{2\sigma_\epsilon}\left((1+h)\bar{\omega}\left[\mu_a + \frac{\theta\sigma_a^2}{3(\sigma_\epsilon + \Delta\omega)}\right] - \omega_L\right)}_{\text{Increase in Volume}} = 0$$
$$\implies \omega_L = -\sigma_\epsilon - \Delta\omega + (1+h)\bar{\omega}\mu_a \tag{20}$$

To get the best response function of the firm in the licensed sector, we rearrange the expression above and substitute in the definition for the net benefit of licensing, $\Delta\omega = (\omega_L - c_0) - (\omega_U + \mu_\epsilon)$:

$$\boxed{\omega_L(\omega_U) = \frac{1}{2}[(1+h)\bar{\omega}\mu_a + \omega_U + c_0 + (\mu_\epsilon - \sigma_\epsilon)]} \tag{21}$$

The best response function for the wages in the licensed sector is increasing in the level of human capital that is bundled with the license h and with the quality of the firm's technology $\bar{\omega}$. It is also increasing in the wage offered by the unlicensed firm, the cost of licensing, and the minimum taste for the unlicensed sector, $\mu_\epsilon - \sigma_\epsilon$.

To find the best response function for Firm 2, we assert that Firm 2 chooses ω_U to maximize its profits, π_2. When we take the first-order condition $\frac{\partial\pi_2}{\partial\omega_U} = 0$, we get

$$\omega_U = -\sigma_\epsilon + \Delta\omega + \bar{\omega}\mu_a \tag{22}$$

To get the best response function of Firm 2, we rearrange the expression above and use the definition for the net benefit of licensing $\Delta\omega = (\omega_L - c_0) - (\omega_U + \mu_\epsilon)$:

$$\boxed{\omega_U(\omega_L) = \frac{1}{2}[\bar{\omega}\mu_a + (\omega_L - c_0) - (\mu_\epsilon + \sigma_\epsilon)]} \tag{23}$$

The best response function for the wages in the unlicensed sector is increasing with the quality of the firm's technology $\bar{\omega}$, the average ability of all workers, and the competing wages in the licensed sector. It is decreasing in the cost of obtaining a license and the maximum taste for the unlicensed sector by workers, $\mu_\epsilon + \sigma_\epsilon$. At the Nash equilibrium, both firms' wages are mutual best responses. Substituting the best response of the firm in the licensed sector into the best response function for the firm in the unlicensed sector, we solve for the equilibrium wage in the unlicensed sector, ω_U^*:

$$\begin{aligned}\omega_U &= \frac{1}{2}[\bar{\omega}\mu_a + -c_0 - (\mu_\epsilon + \sigma_\epsilon)] + \frac{1}{2}\left[\frac{1}{2}[(1+h)\bar{\omega}\mu_a + \omega_U + c_0 + (\mu_\epsilon - \sigma_\epsilon)]\right] \\ \implies &\boxed{\omega_U^* = \left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}c_0 - \frac{1}{3}\mu_\epsilon - \sigma_\epsilon}\end{aligned} \tag{24}$$

To solve for the equilibrium wages in the licensed sector, we insert equilibrium wages from the unlicensed sector into the best response function for the licensed sector:

$$\begin{aligned}\omega_L &= \frac{1}{2}[(1+h)\bar{\omega}\mu_a + c_0 + (\mu_\epsilon - \sigma_\epsilon)] + \frac{1}{2}\left[\left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}c_0 - \frac{1}{3}\mu_\epsilon - \sigma_\epsilon\right] \\ \implies &\boxed{\omega_L^* = \left(1 + \frac{2}{3}h\right)\bar{\omega}\mu_a + \frac{1}{3}c_0 + \frac{1}{3}\mu_\epsilon - \sigma_\epsilon}\end{aligned} \tag{25}$$

To solve for the fraction of licensed workers, we substitute equilibrium wages into the expression for the fraction of licensed workers in Equation (26):

$$f^* = \frac{1}{2} + \frac{\bar{\omega}\mu_a h - c_0 - \mu_\epsilon}{6\sigma_\epsilon}. \tag{26}$$

Defining $\underline{c} \equiv h\bar{\omega}\mu_a - \mu_\epsilon - 3\sigma_\epsilon$, it is straightforward to show that if the average cost of licensing, c_0, is lower than $\underline{c}$, licensing is sufficiently cheap. Then, all workers obtain a license and work in the licensed sector ($f = 1$). Likewise, defining $\bar{c} \equiv h\bar{\omega}\mu_a - \mu_\epsilon + 3\sigma_\epsilon$, if the average cost of licensing, c_0, is higher than $\bar{c}$, licensing is sufficiently onerous. Hence, all workers prefer not to obtain a license ($f = 0$). It is only for intermediate value $c_0 \in (\underline{c}, \bar{c})$ that we observe a nonzero fraction of workers in both the licensed and unlicensed sectors.

We further simplify the expression for the fraction of licensed workers in Equation (26) and the equilibrium wages for workers in equations using the definitions for $\bar{c}$ and $\underline{c}$:

$$\boxed{f^* = \left(\frac{\bar{c} - c_0}{6\sigma_\epsilon}\right)}, \tag{27}$$

$$\boxed{\omega_U^* = \bar{\omega}\mu_a - \frac{1}{3}(c_0 - \underline{c})}, \tag{28}$$

$$\boxed{\omega_L^* = \omega_U^* + \frac{1}{3}h\bar{\omega}\mu_a + \frac{2}{3}(c_0 + \mu_\epsilon)}. \tag{29}$$

Corollary 1. *Wages are unambiguously higher in the licensed sector than in the unlicensed sector, and the cost of licensing is increasing the wedge between these two wages. In equilibrium, unlicensed workers also experience a wage benefit from the human capital that is bundled with the licensing. This wage benefit is half the human capital benefit experienced by licensed workers.*

The fact that licensing is bundled with human capital h increases the market return to licensed labor and, in doing so, increases the value of the outside option of workers who opt not to become licensed. Consistent with this prediction of the model, Han and Kleiner (2016) provide evidence that workers in a licensed occupation who do not possess a license but can practice because of grandfathering provisions experience a 5 percent increase in wages as a result of their occupation becoming licensed, when compared to similar unlicensed workers in occupations with no licensing requirements. By contrast, the wage premium to licensed workers in the occupation, when compared to similar unlicensed workers in occupations with no licensing requirements, is 12 percentage points higher than the wage premium experienced by grandfathered workers.

Corollary 2. *Given two distinct groups of workers, B and W, in which the average cost of licensing is greater for group B than for group W* (i.e., $c_{o,B} > c_{0,W}$)*, unlicensed B workers earn less than unlicensed W workers. By contrast, licensed B workers earn more than licensed W workers,* ceteris paribus*. This follows from the fact that wages are decreasing in c_0 for unlicensed workers (Equation 6) but increasing in c_0 for licensed workers (Equation 7).*

The result of this corollary offers testable predictions. First, unlicensed black men earn less, on average, than unlicensed white men. Second, licensed black men working in occupations with felony restrictions earn, on average, slightly more than licensed white men in similar occupations. The presumption here is that the felony restriction imposes a higher average cost of licensing on black men relative to white men. Using data from the Bureau of Justice Statistics, Sakala (2014) documents that black men are six times more likely to be incarcerated than white men, which is consistent with this assumption.

A.2 Proof of Proposition 3

Proof. By definition, the license premium is

$$\alpha \equiv \frac{\omega_L^* - \omega_U^*}{\omega_U^*} = \frac{\frac{1}{3}\bar{\omega}\mu_a h + \frac{2}{3}(c_0 + \mu_\epsilon)}{\left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}(c_0 + \mu_\epsilon) - \sigma_\epsilon}. \tag{30}$$

The license premium increases in c_0 because the wage gap (numerator) increases in c_0 and the wage in the unlicensed sector (denominator) decreases in c_0. In particular, the derivative of the licensing premium with respect to c_0 is

$$\frac{d\alpha}{dc_0} = \frac{1}{3}\left(\frac{\omega_L - \omega_U}{\omega_U^2}\right) > 0. \tag{31}$$

The derivative of the licensing premium with respect to the mean ability is

$$\frac{d\alpha}{d\mu_a} = -\frac{\bar{\omega}[h(\mu_\epsilon + \sigma_\epsilon + c_0) + 2(c_0 + \mu_\epsilon)]}{3{\omega_U^*}^2} \implies \frac{d\alpha}{d\mu_a} < 0. \tag{32}$$

The derivative of the licensing premium with respect to h is

$$\frac{d\alpha}{dh} = \frac{\bar{\omega}\mu_a[2\omega_U^* - \omega_L^*]}{3{\omega_U^*}^2} \tag{33}$$

Therefore, $\frac{d\alpha}{dh} > 0 \implies 2\omega_U^* - \omega_L^* > 0$, which holds when $\frac{\omega_L^* - \omega_U^*}{\omega_U^*} < 1$ (*i.e.*, $\alpha < 1$).

The positive relationship between the licensing premium and the dispersion in sector taste comes from the fact that wages in the unlicensed sector (denominator) fall with σ_ϵ. $\square$

A.3 Proof of Proposition 4

The total social surplus is the sum of the firm's revenue minus the expected cost of licensing. Since the expected wages of employees are a cost to firms and a benefit to workers, they net out in the social surplus calculation in the case where we place an equal weighting on firm profits and net worker wages:

$$
\begin{aligned}
SS =& \underbrace{(1+h)\bar{\omega}\left(\mu_a + \frac{\theta\sigma_a^2}{3(\sigma_\epsilon + \Delta\omega)}\right)\left(\frac{1}{2} + \frac{\Delta\omega}{2\sigma_\epsilon}\right)}_{\text{Firm 1 Revenue}} + \underbrace{\bar{\omega}\left[\mu_a - \frac{\theta\sigma_a^2}{3(\sigma_\epsilon - \Delta\omega)}\right]\left(\frac{1}{2} - \frac{\Delta\omega}{2\sigma_\epsilon}\right)}_{\text{Firm 2 Revenue}} \\
&- \underbrace{\left[c_0 - \frac{\theta^2\sigma_a^2}{3(\sigma_\epsilon + \Delta\omega)}\right]\left(\frac{1}{2} + \frac{\Delta\omega}{2\sigma_\epsilon}\right)}_{\text{Expected Licensing Costs}} \\
=& \frac{1}{2\sigma_\epsilon}(1+h)\bar{\omega}\left(\mu_a(\sigma_\epsilon + \Delta\omega) + \frac{1}{3}\theta\sigma_a^2\right) + \frac{1}{2\sigma_\epsilon}\bar{\omega}\left(\mu_a(\sigma_\epsilon - \Delta\omega) - \frac{1}{3}\theta\sigma_a^2\right) \\
&- \frac{1}{2\sigma_\epsilon}\left(c_0(\sigma_\epsilon + \Delta\omega) - \frac{1}{3}\theta\sigma_a^2\right)
\end{aligned} \tag{34}
$$

To find the optimal social cost of licensing, we take the derivative of the social surplus with respect to the cost, c_0. Recall the following:

$$
\Delta\omega = \frac{1}{3}(\bar{\omega}\mu_a h - c_0 - \mu_\epsilon) \implies \frac{d\Delta\omega}{dc_0} = -\frac{1}{3} \tag{35}
$$

Therefore,

$$
\begin{aligned}
&\frac{d(SS)}{dc_0} = 0 \\
&\implies -\frac{1}{6\sigma_\epsilon}(1+h)\bar{\omega}\mu_a + \frac{1}{6\sigma_\epsilon}\bar{\omega}\mu_a - \frac{1}{2\sigma_\epsilon}(\sigma_\epsilon + \Delta\omega) + \frac{1}{6\sigma_\epsilon}c_0 = 0 \\
&\implies \boxed{c_0^* = \frac{1}{2}(\bar{c} + h\bar{\omega}\mu_a)}
\end{aligned} \tag{36}
$$